

"Uncertainty Analysis and Control in Nonlinear, Multiscale,
Interconnected Systems"

AFOSR grant FA9550-06-1-0088

Igor Mezić

Department of Mechanical Engineering,
University of California, Santa Barbara

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Abstract

The work under this grant continued our contribution to control of mixing in uncertain, complex environments, energy transfer in nonlinear oscillators and fundamental uncertainty theory for complex systems. Our work on control of mixing, applicable to problems of combustion in microengines, resulted in 3 publications in prominent journals [1, 8, 7].

In this project we developed the theory of energy transfer that leads to synchronization in nonlinear oscillators in several directions. In [3] we have studied effects of asymmetry on mitigating limit cycle oscillations in aeroengines. This work earlier lead to development of a patent and a commercial product in Pratt and Whitney jet engines. In [4], we analyzed the underlying system using nonlinear dynamics tools to elucidate the effect of asymmetry on instabilities in the system. We have also studied of coordinated motion [5, 2] where a large system of units are designed to move under local dynamics (affected only by the unit position and velocity) and positions and velocities of immediate neighbours. The amount of communication required between units in such an arrangement is minimal, but global transitions between desired states of operation are enabled by appropriate design. Alternatively (and similar to the situation in thermoacoustics) transitions of this sort in power systems can not be tolerated since they can lead to global desynchronization. In a study of this, applicable to power system instabilities in military aircraft, we gave conditions for such instability in a model system [10] and developed this into a full-blown theory of instabilities in realistic power systems. We have published a fundamental study on uncertainty analysis in dynamical systems in [9]. The PI has presented on these topics in numerous conference presentations, most notably in an Invited Plenary Lecture at a leading international conference on Dynamical Systems the SIAM Conference on Applications of Dynamical Systems and Opening Lecturer, First Lab on a Chip World Congress, Edinburgh, Scotland (2007). He was also an organizer and participant in the highly selective NSF Workshop on Complex Systems that produced and advisory report to the NSF on the topic, and was a participant in a number of invitation only exploratory workshops organized by AFOSR.

There was an active exchange of information with researchers from the United Technologies Research Center on uncertainty analysis and control of thermoacoustic instabilities [3, 6].

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1 Accomplishments

The key research achievements of the project have been:

1.1 Formational transitions in systems of inertial nonlinear oscillators

Understanding mechanisms for conformation change in large networks of oscillators leads to comprehension of robustness notions in large interconnected dynamical systems. Biological systems are known to be extremely robust to most environmental perturbation while in certain situations they embrace external influence to carry out a particular task. In light of this, the connection with networked or distributed control systems becomes clear. In the paper [4], we study the dynamical properties of energy transfer in a coupled oscillator system undergoing formation change. We use a series of dynamical system tools to identify energy pathways in the system that enable conformation change. We find that during internal resonance, a certain structure appears which channels energy in a manner that enables this formation change to occur.

Networks of nonlinear systems often have different pathways for energy transfer and the degree to which their resistance differs often defines the robustness of the system. A system in which all conduits for energy transfer experience the same resistance are relatively robust to a specific perturbation. On the other hand, systems which have one (or a few) paths in which energy transfer is much easier than the average resistance are deemed non-robust because a specific perturbation on this path will dominate its response. In the paper [4], we call these perturbations structured as their shape (or direction in phase space) are tuned to the path where the resistance to energy transfer is least. The result of perturbation corresponding to a structure in which the system responds greatly may be desirable or not. In this paper we study a system that transitions between two wells of a potential. This can be extended to a case where a specific perturbation is applied and energy is transferred through the structure of the system effectively to accomplish a particular desired task such as formation change for large formations of UAV's or satellites. In figure 1 we show, on the left, the angular position of a

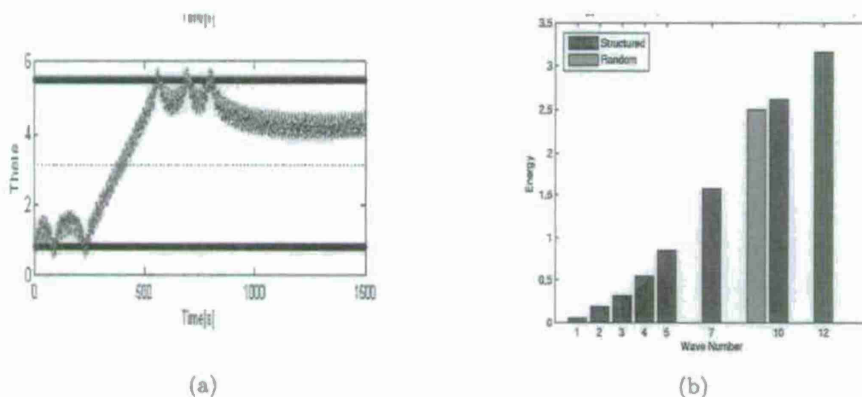


Figure 1: Left: Angular position of a system of oscillators vs. time showing synchronized transition from one direction to another. Right: Structured mode signals can induce transition with high accuracy and low energy. Random signals (in red) are much less effective.

system of oscillators vs. time evolving according to

$$mh^2\ddot{\theta}_k = S(\theta_{k-1} - 2\theta_k + \theta_{k+1}) + 2ahDe^{a(\cos(\theta_k)h-h+x_0)}(e^{a(\cos(\theta_k)h-h+x_0)} - 1)\sin(\theta_k) \quad (1)$$

This can be thought of as a system of n UAV's whose direction in a plane is described by θ_i $i = 1, \dots, n$, coupled only to nearest neighbor, that fly at a constant velocity v in that direction. The plot shows synchronized transition from one direction to another. On the right of the same figure we show the amount of energy contained in a modal perturbation needed to induce a formational transition. It is seen that structured mode signals can induce transition with high accuracy and low energy. Random signals (in red) are much less effective thus indicating systems robustness.

1.2 Exact reduced order models for a system with no scale separation

In the paper [2] we studied the same model for global formation change but with emphasis on multiple-scale behavior and robustness to environmental noise. We show that the dynamical behavior of this model cannot be understood by considering the slowest modes only: there is an inverse cascade by which the effects of changes in small scales are felt by the largest scales and the mean-field closure does not work. Despite this, a one and a half degree of freedom model is derived that includes the influence of the small-scale dynamics and predicts conformational changes accurately. Thus, we provide a reduced model for the system in which there is no separation of scales. Specifically, a low order model that retains the essential mechanism describing the global flipping behavior is developed here. We are guided by the observation that the average angle variables provide a good coarse description of the flipping process. There is, however, no separation of time scales in this system so that simple truncation to a low order model does not retain sufficient dynamics to incorporate flipping. Furthermore, routine averaging methods are not applicable since intrinsic resonances induce coupling on all scales. Our procedure is to write an exact equation for the evolution for the average angle which will necessarily be coupled, and then introduce an approximation that removes the coupling but retains the essential influence of higher order scales in the low dimensional model. The modal behavior of the system is shown on the left in figure 2, and it is clear that nonlinear dynamics is dominant in mode 0 (upper left corner) and behavior close to harmonic oscillations is shown for all the other modes. The relative error in numerically computed flipping time predictions between the 11/2 degree of freedom reduced system and the full 200 degree of freedom system for 500 random initial conditions is shown on the right of figure 2, showing that the error of approximation in this system with no scale separation is small.

In addition we study the efficiency of small-scale perturbations and noise on formational change. It is shown that zipper travelling wave perturbations provide an efficient means for inducing such change. Also, even in a noisy environment the system is able to flip from one formation to another, provided a specific, structured input is given.

1.3 Fast global instabilities in power systems

Consider the network of nonlinear systems that make up aircraft power systems. Clearly, it is desired that these systems are impervious to any perturbation, while it is found in some situations that specific perturbation effects the system in a major and negative way. In [10] we studied a power system consisting of n generators and 1 motor (load), whose configuration is shown on the left of the figure 3

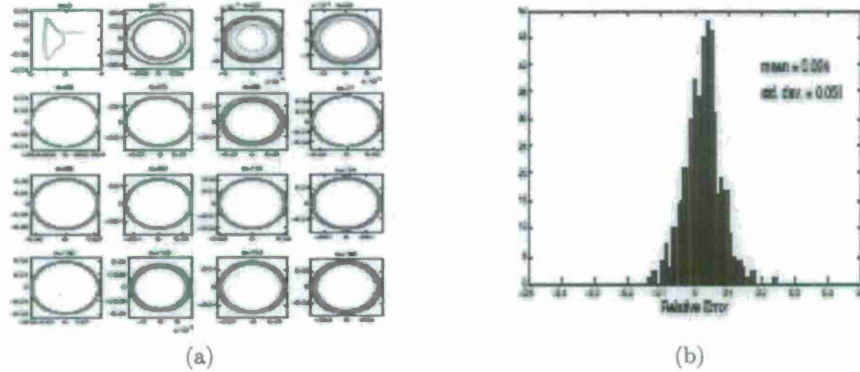


Figure 2: Left: Phase space projections of individual modes of oscillation. Right: Error for prediction of the 11/2 degree of freedom model.

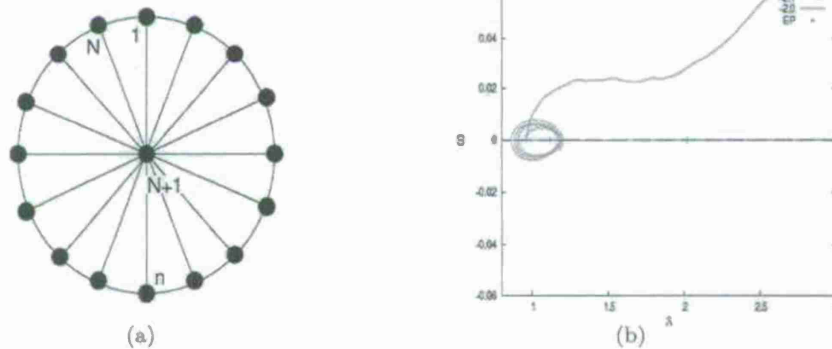


Figure 3: Left: Configuration of the power system studied. Right: Phase space projection: red curve shows desynchronization.

On the right of figure 3, we show the phase space of the collective mode (average desynchronization) of the power system. The blue lines represent trapped motions that show normal (slightly librating) operation, and the red lines denote divergence motions that correspond to instability of power grid. It is seen that, provided a specific initial perturbation is given, the system can go unstable even starting from what seems to be a stable region of the phase space.

2 Personnel supported:

Faculty: Igor Mezić, **Postdoctoral fellow:** George Mathew,

Partially supported graduate students: Thomas John, Bryan Eisenhower.

3 Interactions/transitions:

The PI and other members of the group gave a number of invited lectures on the topics of research described here, for example at Lawrence Livermore National Laboratory, Conference on Applications of Dynamical Systems in Snowbird, AFOSR Robust Decision Making workshop.

There was an interaction with UTRC's Control and Dynamics group on topics in control and uncertainty analysis of thermoacoustic instabilities.

4 Transitions

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Joint work on coupled oscillator models of thermoacoustic instability control concept derived from coupled oscillator models.

5 Honors/Awards

- The PI gave a plenary lecture at the , Second International Conference on Dynamics, Vibration and Control in Beijing, 2006.
- The PI recieved a United Technologies Senior Vice President's Special Award (2007).
- The PI presented Invited Plenary Lecture at a leading international conference on Dynamical Systems the SIAM Conference on Applications of Dynamical Systems
- The PI was the Opening Lecturer, First Lab on a Chip World Congress, Edinburgh, Scotland (2007).

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